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## THE SUMMATION OF TWO SERIES.

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As the following series occur in the solution of problem 121, Calculus, the method of summing them, though neither difficult nor tedious, may be of interest to many of our readers.

$$y = 1 - \frac{n^2 x^2}{2!} + \frac{n^2(n^2+8)x^4}{4!} - \frac{n^2(n^4+40n^2+184)x^6}{6!} + \dots \quad (A).$$

$$y = \frac{nx}{1!} - \frac{n(n^2+2)x^3}{3!} + \frac{n(n^4+20n^2+24)x^5}{5!} - \frac{n(n^6+70n^4+784n^2+720)x^7}{7!} + \dots \quad (B).$$

The general term of both being

$$a_{r+4} = - \frac{r(r+1)_r + [2(r+2)^2 + n^2]a_{r+2}}{(r+3)(r+4)}.$$

$$\text{From (A), } dy/dx = -n^2 x + \frac{n^2(n^2+8)x^3}{3!} - \frac{n^2(n^4+40n^2+184)x^5}{5!} + \dots \quad (1).$$

$$d^2y/dx^2 = -n^2 + \frac{n^2(n^2+8)x^2}{2!} - \frac{n^2(n^4+40n^2+184)x^4}{4!} + \dots \quad (2).$$

$$n^2 \text{ times (A) gives } n^2 y = n^2 - \frac{n^4 x^2}{2!} + \frac{n^4(n^2+8)x^4}{4!} - \dots \quad (3).$$

$$2x(1+x^2) \text{ times (1) gives } 2x(1+x^2)(dy/dx) = -2n^2 x^2 + \frac{2n^2(n^2+2)x^4}{3!} - \dots \quad (4).$$

$(1+x^2)^2$  times (2) gives

$$(1+x^2)^2(d^2y/dx^2) = -n^2 + \frac{n^2(n^2+4)x^2}{2!} - \frac{n^2(n^4+16n^2+16)x^2}{4!} + \dots \quad (5).$$

$$(3) + (4) + (5) \text{ gives } (1+x^2)^2(d^2y/dx^2) + 2x(1+x^2)(dy/dx) + n^2 y = 0 \dots \quad (6).$$

From (B),  $dy/dx =$

$$n - \frac{n(n^2+2)x^2}{2!} + \frac{n(n^4+20n^2+24)x^4}{4!} - \frac{n(n^6+70n^4+784n^2+720)x^6}{6!} + \dots \quad (7).$$

$$d^2y/dx^2 = - \frac{n(n^2+2)x}{1!} + \frac{n(n^4+20n^2+24)x^3}{3!}$$

$$-\frac{n(n^6+70n^4+784n^2+720)x^6}{5!} \dots \dots (8).$$

$n^2$  times (B) gives

$$n^2 y = \frac{n^3 x}{1!} - \frac{n^3(n^2+2)x^3}{3!} + \frac{n^3(n^4+20n^2+24)x^5}{5!} \dots \dots (9).$$

$2x(1+x^2)$  times (7) gives

$$2x(1+x^2)(dy/dx) = 2nx - \frac{2n^3 x^3}{2!} + \frac{3n^3(n^2+8)x^5}{4!} \dots \dots (10).$$

$$(1+x^2)^2(d^2y/dx^2) = -\frac{n(n^2+2)x}{1!} + \frac{n^3(n^2+8)x^3}{3!} - \frac{n^3(n^4+30n^2+104)x^5}{5!} + \dots \dots (11).$$

$$(9) + (10) + (11) \text{ gives } (1+x^2)^2(d^2y/dx^2) + 2x(1+x^2)(dy/dx) + n^2 y = 0, \\ \text{the same as (6)} \dots \dots (12).$$

Multiplying (6) and (12) through by  $2(dy/dx)$ , and integrating, we get  $(1+x^2)^2(dy/dx)^2 + n^2 y^2 + C = 0$ .

When  $x=0$ , from (A),  $y=1$ ,  $dy/dx=0$ .

When  $x=0$ , from (B),  $y=0$ ,  $dy/dx=n$ .

$\therefore$  In either case,  $C=-n^2$ .  $\therefore (1+x^2)^2(dy/dx)^2 = n^2(1-y^2)$ .

$$(1+x^2)dy/dx = \pm n\sqrt{(1-y^2)} \text{ or } \frac{dy}{\sqrt{(1-y^2)}} = \pm \frac{ndx}{1+x^2}.$$

$\therefore \sin^{-1}y = n\tan^{-1}x + D$ , or  $\sin^{-1}y = n\cot^{-1}x + D$ .

From (A), when  $x=0$ ,  $y=1$ .

$$\therefore D = \frac{1}{2}\pi. \quad \therefore \sin^{-1}y = n\tan^{-1}x + \frac{1}{2}\pi = n\cot^{-1}x + \frac{1}{2}\pi.$$

$$\therefore y = \sin(n\tan^{-1}x + \frac{1}{2}\pi) = \sin(n\cot^{-1}x + \frac{1}{2}\pi).$$

$$\therefore y = \cos(n\tan^{-1}x) = \cos(n\cot^{-1}x).$$

From (B), when  $x=0$ ,  $y=0$ .

$$\therefore D = 0. \quad \therefore \sin^{-1}y = n\tan^{-1}x = n\cot^{-1}x.$$

$$\therefore y = \sin(n\tan^{-1}x) = \sin(n\cot^{-1}x).$$